Exercise 7 to the lectures on **Relativity**

J. Jersák, WS 2000/2001

Problem 20: Mandelstam Variables (1 point)

For the 4-particle scattering process

$$p_a + p_b = p_d + p_f \tag{1}$$

consider the Mandelstam variable $u = c^2(p_a - p_f)^2$. Calculate the sum $s + t + u$ and discuss how many Mandelstam variables are independent.

Problem 21: Compton Scattering (2 points)

Consider the elastic head on collision of a low energy photon from the cosmic microwave background with energy $E_\gamma = 3 \times 10^{-4}\ eV$ with a high energy cosmic ray proton of energy $E_p = 10^{11} m_p c^2$.

a. Calculate the dependence of the photon energy E'_γ on E_γ and E_p, if γ is scattered exactly in backward direction,

b. and determine the numerical value of E'_γ.

Problem 22: Relativistic Kinematics (*2 points)

Discuss, under which kinematic conditions on the initial state

a. the creation of one additional electron positron pair is possible, if in the laboratory frame a single photon is scattered on a single electron. (The final state contains 2 electrons and a positron.)

b. a collision of two protons with the same energy E_p can produce a final state of two protons and one additional particle d with rest mass m_d.

Problem 23: Relativistic Billiards (*3 points)

Two billiard balls a and b of equal rest mass m scatter elastically. In the laboratory system S^L a is at rest and b hits a with velocity v and $\vec{v} = v\vec{e}_1$. Let θ_a^L and θ_b^L be angles of a and b velocities after collision relative to $\vec{e}_1$.

a. Lorentz transform the scattering angles θ_a^{CM} and θ_b^{CM} in S^{CM} to S^L (use 4-velocities).

b. Calculate $tan\theta_a^L\ tan\theta_b^L$.

c. Using this result show, that in S^L the opening angle $\theta_a^L + \theta_b^L$ is smaller than the nonrelativistic result. What is the nonrelativistic result ?

Problem 24: Boost Commutator (*2 points)

For two infinitesimal Lorentz boosts $L_1 = L(\delta v_1 \vec{e}_1, \vec{0})$ and $L_2 = L(\delta v_2 \vec{e}_2, \vec{0})$

a. calculate the commutator $[L_1, L_2] = L_1 L_2 - L_2 L_1$ up to order $\mathcal{O}(\delta v_1 \delta v_2)$.

b. to which infinitesimal transformation $L = L(\delta \vec{v}, \delta \vec{\phi})$ does this result correspond ?

Exercises will be collected on: **Thu 8.2.01, Kl. Phys., 02.00 pm**,

Exercise lead by T. Neuhaus (date not yet known)

* voluntary problem